UNDERSTANDING ENGINEERING MATHEMATICS

JOHN BIRD

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Understanding Engineering Mathematics

Why is knowledge of mathematics important in engineering?

A career in any engineering or scientific field will require both basic and advanced mathematics. Without mathematics to determine principles, calculate dimensions and limits, explore variations, prove concepts, and so on, there would be no mobile telephones, televisions, stereo systems, video games, microwave ovens, computers, or virtually anything electronic. There would be no bridges, tunnels, roads, skyscrapers, automobiles, ships, planes, rockets or most things mechanical. There would be no metals beyond the common ones, such as iron and copper, no plastics, no synthetics. In fact, society would most certainly be less advanced without the use of mathematics throughout the centuries and into the future.

Electrical engineers require mathematics to design, develop, test or supervise the manufacturing and installation of electrical equipment, components, or systems for commercial, industrial, military or scientific use.

Mechanical engineers require mathematics to perform engineering duties in planning and designing tools, engines, machines and other mechanically functioning equipment; they oversee installation, operation, maintenance and repair of such equipment as centralised heat, gas, water and steam systems.

Aerospace engineers require mathematics to perform a variety of engineering work in designing, constructing and testing aircraft, missiles and spacecraft; they conduct basic and applied research to evaluate adaptability of materials and equipment to aircraft design and manufacture and recommend improvements in testing equipment and techniques.

Nuclear engineers require mathematics to conduct research on nuclear engineering problems or apply principles and theory of nuclear science to problems concerned with release, control and utilisation of nuclear energy and nuclear waste disposal.

Petroleum engineers require mathematics to devise methods to improve oil and gas well production and determine the need for new or modified tool designs; they oversee drilling and offer technical advice to achieve economical and satisfactory progress.

Industrial engineers require mathematics to design, develop, test and evaluate integrated systems for managing industrial production processes, including human work factors, quality control, inventory control, logistics and material flow, cost analysis and production coordination.

Environmental engineers require mathematics to design, plan or perform engineering duties in the prevention, control and remediation of environmental health hazards, using various engineering disciplines; their work may include waste treatment, site remediation or pollution control technology.

Civil engineers require mathematics in all levels in civil engineering – structural engineering, hydraulics and geotechnical engineering are all fields that employ mathematical tools such as differential equations, tensor analysis, field theory, numerical methods and operations research.

Knowledge of mathematics is therefore needed by each of the engineering disciplines listed above.

It is intended that this text – *Understanding Engineering Mathematics* – will provide a step-by-step approach to learning all the fundamental mathematics needed for your engineering studies.

To Sue

Understanding Engineering Mathematics

John Bird, BSc (Hons), CEng, CMath, CSci, FIMA, FIET, FCollT

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Preface

Studying engineering, whether it is mechanical, electrical, aeronautical, communications, civil, construction or systems engineering, relies heavily on an understanding of mathematics. In fact, it is not possible to study any engineering discipline without a sound knowledge of mathematics. What happens, then, when a student realises he/she is very weak at mathematics – an increasingly common scenario? The answer may hopefully be found in this textbook ***Understanding Engineering Mathematics*** which explains as simply as possible the steps needed to become better at mathematics and hence gain real confidence and understanding in their chosen engineering subject.

Understanding Engineering Mathematics is an amalgam of three books – *Basic Engineering Mathematics*, *Engineering Mathematics* and *Higher Engineering Mathematics*, all currently published by Routledge. The point about ***Understanding Engineering Mathematics*** is that it is all-encompassing. We do not have to think 'what course does this book apply to?'. The answer is that it encompasses all courses that include some engineering content in their syllabus, from beginning courses up to degree level.

The primary aim of the material in this text is to provide the fundamental analytical and underpinning knowledge and techniques needed to successfully complete scientific and engineering principles modules covering a wide range of programmes. The material has been designed to enable students to use techniques learned for the analysis, modelling and solution of realistic engineering problems. It also aims to provide some of the more advanced knowledge required for those wishing to pursue careers in a range of engineering disciplines. In addition, the text will be suitable as a valuable reference aid to practising engineers.

In ***Understanding Engineering Mathematics***, theory is introduced in each chapter by a full outline of essential definitions, formulae, laws, procedures, etc. The theory is kept to a minimum, for problem solving is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.

The material has been ordered into the following **fourteen convenient categories**: number and algebra, further number and algebra, areas and volumes, graphs, geometry and trigonometry, complex numbers, matrices and determinants, vector geometry, differential calculus, integral calculus, differential equations, statistics and probability, Laplace transforms and Fourier series. Each topic considered in the text is presented in a way that assumes in the reader very little previous knowledge.

With a plethora of engineering courses worldwide it is not possible to have a definitive ordering of material; it is assumed that both students and instructors/lecturers alike will 'dip in' to the text according to their particular course structure.

The text contains some **1500 worked problems**, **2750 further problems** (with answers), arranged within **370 Exercises**, **255 multiple choice questions arranged into 9 tests**, **34 Revision Tests**, **750 line diagrams** and **14 lists of formulae/revision hints**.

Worked solutions to all 2750 further problems have been prepared and can be **accessed free via the publisher's website** (see below).

At intervals throughout the text are some **34 Revision Tests** to check understanding. For example, Revision Test 1 covers the material in Chapters 1 and 2, Revision Test 2 covers the material in Chapters 3 to 5, Revision Test 3 covers the material in Chapters 6 to 8, and so on.

'Learning by example' is at the heart of *Understanding Engineering Mathematics*.

JOHN BIRD
Defence School of Marine Engineering
HMS *Sultan*, formerly
University of Portsmouth and Highbury
College, Portsmouth

Free web downloads via
http://www.routledge.com/cw/bird

Worked Solutions to Exercises

Within the text are some 2750 further problems arranged within 370 Exercises. Worked solutions have been prepared and can be accessed free by students and staff.

Instructor's manual

This provides full worked solutions and mark scheme for all 34 Revision Tests in this book. The material is available to lecturers/instructors only.

Illustrations

Lecturers can download electronic files for all 750 illustrations within the text.

Famous Mathematicians/Engineers

From time to time in the text, some 38 famous mathematicians/engineers are referred to and emphasised with an asterisk*. Background information on each of these is available via the website.

Mathematicians/Engineers involved are: **Argand, Bessel, Boole, Boyle, Cauchy, Celsius, Charles, Cramer, de Moivre, de Morgan, Descartes, Euler, Fourier, Frobenius, Gauss, Hooke, Karnaugh, Kirchhoff, Kutta, Laplace, Legendre, Leibniz, L'Hopital, Maclaurin, Napier, Newton, Ohm, Pappus, Pascal, Poisson, Pythagoras, Raphson, Rodrigues, Runge, Simpson, Taylor, Wallis and Young.**

John Bird is the former Head of Applied Electronics in the Faculty of Technology at Highbury College, Portsmouth, UK. More recently, he has combined freelance lecturing at the University of Portsmouth with Examiner responsibilities for Advanced Mathematics with City and Guilds, and examining for the International Baccalaureate Organisation. He is the author of over 120 textbooks on engineering and mathematical subjects, with worldwide sales of one million copies. He is currently a Senior Training Provider at the Defence School of Marine Engineering in the Defence College of Technical Training at HMS *Sultan*, Gosport, Hampshire, UK.

Section A

Number and Algebra

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Chapter 1

Basic arithmetic

Why it is important to understand: **Basic arithmetic**

Being numerate, i.e. having an ability to add, subtract, multiply and divide whole numbers with some confidence, goes a long way towards helping you become competent at mathematics. Of course electronic calculators are a marvellous aid to the quite complicated calculations often required in engineering; however, having a feel for numbers 'in our head' can be invaluable when estimating. Do not spend too much time on this chapter because we deal with the calculator later; however, try to have some idea how to do quick calculations in the absence of a calculator. You will feel more confident in dealing with numbers and calculations if you can do this.

At the end of this chapter, you should be able to:

- understand positive and negative integers
- add and subtract whole numbers
- multiply and divide two integers
- multiply numbers up to 12×12 by rote
- determine the highest common factor from a set of numbers
- determine the lowest common multiple from a set of numbers
- appreciate the order of precedence when evaluating expressions
- understand the use of brackets in expressions
- evaluate expressions containing $+$, $-$, $\times$, $\div$ and brackets

1.1 Introduction

Whole numbers are simply the numbers 0, 1, 2, 3, 4, 5 ... (and so on). **Integers** are like whole numbers, but they also include negative numbers. $+3, +5$ and $+72$ are examples of positive integers; $-13, -6$ and -51 are examples of negative integers. Between positive and negative integers is the number 0, which is neither positive nor negative.

The four basic arithmetic operators are add ($+$), subtract ($-$), multiply ($\times$) and divide ($\div$).

It is assumed that adding, subtracting, multiplying and dividing reasonably small numbers can be achieved without a calculator. However, if revision of this area is needed then some worked problems are included in the following sections.

When **unlike signs** occur together in a calculation, the overall sign is **negative**. For example,

$$5 + (-2) = 5 + -2 = 5 - 2 = \mathbf{3}$$
$$3 + (-4) = 3 + -4 = 3 - 4 = \mathbf{-1}$$

and

$$(+5) \times (-2) = \mathbf{-10}$$

Like signs together give an overall **positive sign**. For example,

$$3 - (-4) = 3 - -4 = 3 + 4 = \mathbf{7}$$

and

$$(-6) \times (-4) = \mathbf{+24}$$

1.2 Revision of addition and subtraction

You can probably already add two or more numbers together and subtract one number from another. However, if you need a revision then the following worked problems should be helpful.

Problem 1. Determine 735 + 167

$$\begin{array}{r} \textbf{H T U} \\ 7\ 3\ 5 \\ +\ 1\ 6\ 7 \\ \hline 9\ 0\ 2 \\ \hline 1\ 1\ \ \end{array}$$

(i) 5 + 7 = 12. Place 2 in units (U) column. Carry 1 in the tens (T) column.

(ii) 3 + 6 + 1 (carried) = 10. Place the 0 in the tens column. Carry the 1 in the hundreds (H) column.

(iii) 7 + 1 + 1 (carried) = 9. Place the 9 in the hundreds column.

Hence, **735 + 167 = 902**

Problem 2. Determine 632 − 369

$$\begin{array}{r} \textbf{H T U} \\ 6\ 3\ 2 \\ -\ 3\ 6\ 9 \\ \hline 2\ 6\ 3 \\ \hline \end{array}$$

(i) 2 − 9 is not possible; therefore change one ten into ten units (leaving 2 in the tens column). In the units column, this gives us 12 − 9 = 3

(ii) Place 3 in the units column.

(iii) 2 − 6 is not possible; therefore change one hundred into ten tens (leaving 5 in the hundreds column). In the tens column, this gives us 12 − 6 = 6

(iv) Place the 6 in the tens column.

(v) 5 − 3 = 2

(vi) Place the 2 in the hundreds column.

Hence, **632 − 369 = 263**

Problem 3. Add 27, −74, 81 and −19

This problem is written as 27 − 74 + 81 − 19

Adding the positive integers:	27
	81
Sum of positive integers is	108
Adding the negative integers:	74
	19
Sum of negative integers is	93
108 + −93 = 108 − 93 and taking the sum of the negative integers from the sum of the positive integers gives	108
	−93
	15

Thus, **27 − 74 + 81 − 19 = 15**

Problem 4. Subtract −74 from 377

This problem is written as 377 − −74. Like signs together give an overall positive sign, hence

$$377 - -74 = 377 + 74 \qquad \begin{array}{r} 3\,7\,7 \\ +\ \ 7\,4 \\ \hline 4\,5\,1 \\ \hline \end{array}$$

Thus, **377 − −74 = 451**

Problem 5. Subtract 243 from 126

The problem is 126 − 243. When the second number is larger than the first, take the smaller number from the larger and make the result negative. Thus,

$$126 - 243 = -(243 - 126) \qquad \begin{array}{r} 2\,4\,3 \\ -\ 1\,2\,6 \\ \hline 1\,1\,7 \\ \hline \end{array}$$

Thus, **126 − 243 = −117**

Problem 6. Subtract 318 from −269

The problem is −269 − 318. The sum of the negative integers is

$$\begin{array}{r} 2\,6\,9 \\ +\,3\,1\,8 \\ \hline 5\,8\,7 \\ \hline \end{array}$$

Thus, **−269 − 318 = −587**

Now try the following Practice Exercise

Practice Exercise 1 Further problems on addition and subtraction (answers on page 1108)

In Problems 1 to 15, determine the values of the expressions given, without using a calculator.

1. 67 kg − 82 kg + 34 kg
2. 73 m − 57 m
3. 851 mm − 372 mm
4. 124 − 273 + 481 − 398
5. £927 − £114 + £182 − £183 − £247
6. 647 − 872
7. 2417 − 487 + 2424 − 1778 − 4712
8. −38419 − 2177 + 2440 − 799 + 2834
9. £2715 − £18250 + £11471 − £1509 + £113274
10. 47 + (−74) − (−23)
11. 813 − (−674)
12. 3151 − (−2763)
13. 4872 g − 4683 g
14. −23148 − 47724
15. $53774 − $38441

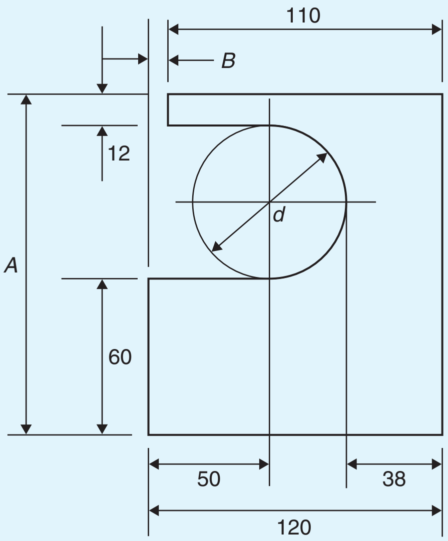

Figure 1.1

16. Figure 1.1 shows the dimensions of a template in millimetres. Calculate the diameter d and dimensions A and B for the template.

1.3 Revision of multiplication and division

You can probably already multiply two numbers together and divide one number by another. However, if you need a revision then the following worked problems should be helpful.

Problem 7. Determine 86 × 7

$$\begin{array}{rr} & \text{H T U} \\ & 8\ 6 \\ \times & 7 \\ \hline & 6\ 0\ 2 \\ \hline & 4\ \ \end{array}$$

(i) 7 × 6 = 42. Place the 2 in the units (U) column and 'carry' the 4 into the tens (T) column.

(ii) 7 × 8 = 56; 56 + 4 (carried) = 60. Place the 0 in the tens column and the 6 in the hundreds (H) column.

Hence, **86 × 7 = 602**

A good grasp of **multiplication tables** is needed when multiplying such numbers; a reminder of the multiplication table up to 12 × 12 is shown on page 6. Confidence with handling numbers will be greatly improved if this table is memorised.

Problem 8. Determine 764 × 38

$$\begin{array}{rr} & 7\,6\,4 \\ \times & 3\,8 \\ \hline & 6\,1\,1\,2 \\ & 2\,2\,9\,2\,0 \\ \hline & 2\,9\,0\,3\,2 \\ \hline \end{array}$$

(i) 8 × 4 = 32. Place the 2 in the units column and carry 3 into the tens column.

(ii) 8 × 6 = 48; 48 + 3 (carried) = 51. Place the 1 in the tens column and carry the 5 into the hundreds column.